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LETTER TO THE EDITOR

A photon rest mass and energy transport in cold plasmas

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Abstract. The energy densities and generalized Poynting vectors for longitudinal and transverse waves in cold plasmas are investigated.

In recent letters (Burman 1972a, b), the author used the Proca equations to discuss the dispersion and absorption of longitudinal electric waves in cold plasmas. The purpose of the present letter is to investigate the energy carried by such waves; in particular, a result obtained by Gertsenshtein (1971) for free space propagation is generalized to the case of propagation in a cold plasma.

Let $\boldsymbol{E}$ and $\boldsymbol{H}$ denote the electric and magnetic fields while ϕ and $\boldsymbol{A}$ denote the scalar and vector potentials. Also, let $c \equiv (\mu_0\epsilon_0)^{-1/2}$, μ_0 and ϵ_0 being the permittivity and permeability of free space, and write the photon rest mass m as $\hbar\omega_c/c^2$, $\hbar$ being Planck's constant divided by 2π. In CGS units, Proca's equations give (Goldhaber and Nieto 1971)

$$8\pi W = \boldsymbol{E}^2+\boldsymbol{H}^2+(\omega_c/c)^2(\phi^2+\boldsymbol{A}^2) \tag{1}$$

and

$$(4\pi/c)\boldsymbol{P} = \boldsymbol{E}\times\boldsymbol{H}+(\omega_c/c)^2\phi\boldsymbol{A} \tag{2}$$

for the electromagnetic energy density W and the generalized Poynting vector $\boldsymbol{P}$. A time factor $\exp(\mathrm{i}\omega t)$ will be taken and $\boldsymbol{\epsilon}$ will denote a unit vector in the direction of propagation; n will denote the refractive index. The quantities $\overline{W}/|\boldsymbol{E}|^2$ and $\overline{\boldsymbol{P}}/|\boldsymbol{E}|^2$, where $\overline{W}$ and $\overline{\boldsymbol{P}}$ are the time-averaged values of W and $\boldsymbol{P}$, will be denoted by ψ and $\boldsymbol{Q}$.

For longitudinal waves $\boldsymbol{H} = 0$, $\boldsymbol{E} = (\mathrm{i}\omega/c)\,(n^2-1)\boldsymbol{A}$ and $\phi\boldsymbol{\epsilon} = n\boldsymbol{A}$; also (Burman 1972b) $n^2 = \{1-(\omega_c/\omega)^2\}(1-X/U)^{-1}$ where $X \equiv \omega_p{}^2/\omega^2$ and $U \equiv 1-\mathrm{i}\nu/\omega$, in which ω_p is the angular plasma frequency and ν is the effective electron–heavy particle collision frequency. Hence

$$16\pi\psi = 1+\left|\frac{\omega^2}{\omega_c{}^2}\left(1-\frac{X}{U}\right)^2-\left(1-\frac{X}{U}\right)\right|+\frac{\omega^2}{\omega_c{}^2}\left|1-\frac{X}{U}\right|^2 \tag{3}$$

and

$$\frac{8\pi}{c}\boldsymbol{Q} = \boldsymbol{\epsilon}\frac{\omega^2}{\omega_c{}^2}\,\mathrm{Re}\left\{\left(1-\frac{X}{U}\right)^{3/2}\left(1-\frac{X}{U}-\frac{\omega_c{}^2}{\omega^2}\right)^{1/2}\right\}. \tag{4}$$

For transverse waves $\boldsymbol{H} = -(\mathrm{i}\omega n/c)\boldsymbol{\epsilon}\times\boldsymbol{A}$, $\boldsymbol{E} = -(\mathrm{i}\omega/c)\boldsymbol{A}$, $\phi = 0$ and $n^2 = 1-X/U-\omega_c{}^2/\omega^2$. Hence

$$16\pi\psi = 1+\left|1-\frac{X}{U}-\frac{\omega_c{}^2}{\omega^2}\right|+\frac{\omega_c{}^2}{\omega^2} \tag{5}$$

and

$$\frac{8\pi}{c}Q = \boldsymbol{\epsilon}\,\mathrm{Re}\left(1-\frac{X}{U}-\frac{\omega_c^2}{\omega^2}\right)^{1/2}. \tag{6}$$

In the following, electron–heavy particle collisions will be neglected. For longitudinal waves n is zero when $\omega = \omega_0$ where $\omega_0^2 \equiv \omega_p^2+\omega_c^2$, infinite when $\omega = \omega_p$ and purely imaginary, corresponding to evanescent disturbances, for $\omega_p < \omega < \omega_0$. Transverse waves are 'cut-off' for $\omega < \omega_0$.

Equations (3) and (4) give

$$16\pi\psi\begin{cases} = \dfrac{2\omega^2}{\omega_c^2}(1-X)^2+X & \omega < \omega_p \text{ and } \omega > \omega_0 \qquad (7a) \\ = 2-X & \omega_p < \omega < \omega_0 \qquad (7b)\end{cases}$$

and

$$\frac{8\pi}{c}Q\begin{cases} = \boldsymbol{\epsilon}(\omega^2-\omega_p^2)^{3/2}(\omega^2-\omega_0^2)^{1/2}/\omega^2\omega_c^2 & \omega > \omega_0 \qquad (8a) \\ = \mathbf{0} & \omega_p < \omega < \omega_0 \qquad (8b) \\ = \boldsymbol{\epsilon}(\omega_p^2-\omega^2)^{3/2}(\omega_0^2-\omega^2)^{1/2}/\omega^2\omega_c^2 & \omega < \omega_p. \qquad (8c)\end{cases}$$

Equation (5) gives $16\pi\psi = 2-X$ or $X+2\omega_c^2/\omega^2$ according as $\omega > \omega_0$ or $\omega < \omega_0$, while (6) gives $(8\pi/c)Q = \boldsymbol{\epsilon}(1-\omega_0^2/\omega^2)^{1/2}$ or $\mathbf{0}$ according as $\omega > \omega_0$ or $\omega < \omega_0$.

Let subscripts l and t denote quantities pertaining to longitudinal and transverse waves, respectively. From the above results

$$\frac{\psi_l}{\psi_t}\begin{cases} = \dfrac{2(\omega^2-\omega_p^2)/\omega_c^2+\omega_p^2}{2\omega^2-\omega_p^2} & \omega > \omega_0 \qquad (9a) \\ = \dfrac{2\omega^2-\omega_p^2}{\omega_p^2+2\omega_c^2} & \omega_p < \omega < \omega_0 \qquad (9b) \\ = \dfrac{2(\omega^2-\omega_p^2)/\omega_c^2+\omega_p^2}{\omega_p^2+2\omega_c^2} & \omega < \omega_p \qquad (9c)\end{cases}$$

and

$$\frac{Q_l}{Q_t} = \frac{(\omega^2-\omega_p^2)^{3/2}}{\omega\omega_c^2} \qquad \omega > \omega_0. \tag{10}$$

If $\omega^2 \gg \omega_0^2$, then $\psi_l/\psi_t \simeq (\omega/\omega_c)^2 \simeq Q_l/Q_t$; for propagation in free space, this holds exactly (Gertsenshtein 1971). If $\max(\omega^2, \omega_c^2) \ll \omega_p^2$, then $\psi_l/\psi_t \simeq 2(\omega_p/\omega_c)^2$.

Gertsenshtein (1971) suggested that the events detected by Weber might be caused by longitudinal electric waves. The frequency f_W in Weber's experiments is $1{\cdot}66\times10^3$ s^{-1}; this is close to the mean interstellar plasma frequency f_p, which is probably about 3×10^2 to 10^3 s^{-1} (Burman 1972b). If f_p is near 10^3 s^{-1}, then the calculations of Gertsenshtein, based on free space propagation, are inapplicable for $f = f_W$, and the cold plasma model treated here also fails. If f_p is near 3×10^2 s^{-1}, then Gertsenshtein's calculations are approximately applicable for $f = f_W$, although, because of variations in the interstellar electron density, the wave will cross regions in which they are inapplicable and also regions in which the cold plasma model fails.

For frequencies well below f_p but exceeding a few tens of cycles per second, Gertsenshtein's calculations are not applicable but the results obtained here are. For frequencies of a few tens of cycles per second and below, the effects of the interstellar magnetic field and of ion motion could be significant.

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